# Introduction to Equivariant Stable Homotopy Theory

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Fix a compact Lie group $G$ and a complete $G$-universe $\mathcal{U}$. The main goal of this talk is to give an introduction to equivariant stable homotopy theory. We introduce the model of the Lewis-May spectra $G\mathscr{S}\mathcal{U}$ today and we will orthogonal $G$-spectra next week. After the point-set model is built, we will introduce the various fixed points constructions. We will also introduce the Wirthmüller isomorphism, the isotropy separation sequence and the Tate diagram.

## 1 The category $G\mathscr{S}U$

In this talk, we will introduce the Lewis-May spectra. We will see orthogonal $G$-spectra next week. A $G$-universe $\mathcal{U}$ is an infinite dimensional $G$-representation such that $\mathcal{U}$ contains the trivial representation and that if $\mathcal{U}$ contains an irreducible representation $V$, $\mathcal{U}$ must contain infinitely many copies of $V$.

**Definition 1.1.** A $G$-prespectrum assigns a pointed $G$-space $E(V)$ for each representation $V$ in the universe $\mathcal{U}$, together with $G$-equivariant structure maps

$$\sigma_{V,W} : S^{W-V} \wedge E(V) \to E(W)$$

whenever $V \subset W \subset \mathcal{U}$, where $W - V$ denotes the orthogonal complement of $V$ in $W$ and $S^{W-V}$ is the one-point compactification of $W - V$. The structure maps satisfy some compatibility conditions. The morphisms are defined to be those maps compatible with structure maps. We use $G\mathcal{P}\mathcal{U}$ to denote the category of $G$-prespectra indexed on the universe $\mathcal{U}$.

**Definition 1.2.** A $G$-spectrum $E$ is a $G$-prespectrum such that all the adjoints $\tilde{\sigma}_{V,W} : E(V) \to \Omega^{W-V}E(W)$ of $\sigma_{V,W}$ are $G$-homeomorphisms. We use $G\mathscr{S}\mathcal{U}$ to denote the category of $G$-spectra indexed on the universe $\mathcal{U}$.

There is the spectrification functor $L$ left adjoint to the inclusion functor $\ell$:

$$L : G\mathscr{S}\mathcal{U} \rightleftarrows G\mathscr{P}\mathcal{U} : \ell$$

We have change of universe functors. For a $G$-equivariant linear isometry $f : \mathcal{U} \to \mathcal{U}'$ (see [EHCT, P137])

$$f_* : G\mathscr{S}\mathcal{U} \rightleftarrows G\mathscr{S}\mathcal{U}' : f^*$$

A map of $G$-spectra $f : E \to E'$ is a weak equivalence if $E(V) \to E'(V)$ is a weak equivalence for all $V$. When $\mathcal{U}$ and $\mathcal{U}'$ are isomorphic $G$-representations, $\mathscr{L}(\mathcal{U},\mathcal{U}')$ is $G$-contractible, and different choices of $f$ will induce weakly equivalent functors $G\mathscr{S}\mathcal{U} \to G\mathscr{S}\mathcal{U}'$.

When $\mathcal{U}$ contains all irreducible $G$-representations, we say it is a complete universe; when $\mathcal{U}$ contains only the trivial representations, we say it is a trivial universe. The $G$-spectra indexed on a complete universe is often called genuine $G$-spectra and on a trivial universe is often called naive $G$-spectra. We assume $\mathcal{U}$ to be a complete universe in this notes, unless otherwise stated. Then $i : \mathcal{U}^G \to \mathcal{U}$ induces an adjunction $(i_*, i^*)$ between naive $G$-spectra and genuine $G$-spectra.

We may use $f_*$ to define the smash product. The idea is to first define the external smash product, which lands in $G\mathscr{S}(U \oplus U)$, and then change the universe back to $\mathcal{U}$. The smash product is unital, associative and commutative up to equivalence.

We skip the details of the following construction.

- We can define suspension spectra functor $\Sigma_G^\infty$; we have sphere spectra $S^n \in G\mathscr{S}\mathcal{U}$ for all integers $n$.
- We also have function spectra $F(E,F) \in G\mathscr{S}\mathcal{U}$ for $E, F \in G\mathscr{S}\mathcal{U}$.
- We have the notion of $G$-CW spectra and we have the $G$-CW approxmiation functor $\Gamma$. For $G$-spectra $E, F$, we use $[E,F]_G$ to denote the homotopy classes of maps $\Gamma E \to \Gamma F$.

**Definition 1.3.** We define the $H$-equivariant homotopy groups of a $G$-spectrum $E$ to be

$$\pi_n^H(E) = [G/H_+ \wedge S^n, E]_G.$$

This assembles into a coefficient system

$$\underline{\pi}_n(E) : \mathscr{O}_G^{op} \to \mathrm{Ab}, \underline{\pi}_n(E)(G/H) = \pi_n^H(E).$$

**Theorem 1.4.** *([EHCT, XII.6.8]) For a map of $G$-spectra $f : E \to E'$, $f(V)$ induces a weak equivalence for all $V$ if and only if $\pi_n^H(f)$ is an isomorphism for all $n \in \mathbb{Z}$ and $H \subset G$.*

We conclude by defining the fixed point spectra. Note that in the Lewis-May model, every object is fibrant, so the fixed point functor is homotopical. In other models, there is usually a "naive fixed point functor" and a "derived fixed point functor". Let $D \in G\mathscr{S}\mathcal{U}^G$ be a naive $G$-spectrum. We define $D^G \in \mathscr{S}\mathcal{U}^G$ to be $D^G(V) = D(V)^G$. For genuine $G$-spectrum $E$, we define $E^G = (i^*E)^G$. We have the following adjunctions between spectra, naive $G$-spectra and genuine $G$-spectra

$$\mathscr{S}\mathcal{U}^G \underset{(-)^G}{\overset{j^*}{\rightleftarrows}} G\mathscr{S}\mathcal{U}^G \underset{i^*}{\overset{i_*}{\rightleftarrows}} G\mathscr{S}\mathcal{U}$$

For naive $G$-spectra $D \in G\mathscr{SU}^G$, we define $D/G \in \mathscr{SU}^G$ to be $L$ applied to levelwise $G$-orbits. It is left adjoint to $j^* = (-)^{triv} : \mathscr{SU}^G \to G\mathscr{SU}^G$. However, for genuine $G$-spectra $E \in G\mathscr{SU}$, the $i^*E/G$ is not a useful definition for $E/G$. We have a substitute definition of $E/G$ when $E$ is $G$-free: in this case, we have a $G$-free naive $G$-spectrum $D$ up to equivalence such that $i_* D \simeq E$. We define $E/G$ to be $D/G$ instead.

For a $G$-spectra $E$, we define the $E$-(co)homology groups of $X$ to be

$$E^n_G(X) = \pi_n(E \wedge X)^G;$$
$$E^G_n(X) = \pi_n F(X, E)^G.$$

## 1.1 Constructions of the transfer map

In this section, we assume the group $G$ is a finite group. Given subgroups $K \subseteq H \subseteq G$, we have a natural map of left cosets $G/K \to G/H$ given by projection. This map lives in the category $\mathbf{Top}^G$ of $G$-spaces. The transfer map is a "wrong way" map $G/H \to G/K$ that lives in the stable homotopy category, that is, a map in $\mathrm{colim}_{V \subset \mathcal{U}}[\Sigma^V_+ G/H, \Sigma^V_+ G/K]_G$.

Let's choose an $H$-representation $W$ and an $H$-equivariant embedding

$$j : H/K \hookrightarrow W$$

such an embedding is completely determined by the image $w := j(H)$. Without loss of generality, we can assume the open unit balls around the image points $g \cdot w$ are pairwise disjoint. Therefore, we get an embedding:

$$H/K_+ \wedge D(W) \to W$$

The Pontryagin-Thom collaspse map (i.e. sending the complement of the open balls $j(H/K_+ \wedge D(W))$ to the point at infinity) gives us a map:

$$S^W \to H/K_+ \wedge S^W$$

then compose with $G_+ \wedge_H (-)$, we get the desired transfer map:

$$tr^H_K : G/H_+ \wedge S^W \to G/K_+ \wedge S^W$$

which represents a map in the equivariant Spanier-Whitehead category $\mathcal{SW}^G$. Note that in the construction of the transfer $tr^H_K$, we made certain choice of the embedding $H/K \hookrightarrow W$. We should remark that different choices would lead to the same class in the Spanier-Whitehead category $\mathcal{SW}^G$.

**Example 1.5.** Let $G = H = C_3$ and $K = \{e\}$ be the trivial subgroup. We let $W$ be a $C_3$-representation of $\mathbb{C}$ by a rotation of $\frac{2}{3}\pi$ around $0$. We embed $C_3/\{e\}$ into $W$ as $2$, $2e^{\frac{2}{3}\pi i}$ and $2e^{\frac{4}{3}\pi i}$. Then the transfer map in this example is a map:

$$tr^{C_3}_{\{e\}} : S^W \to (C_3)_+ \wedge S^W$$

**Consequence:** Using the transfer map, we are able to get covariant functors $\pi_n(E) : \mathscr{O}_G \to \mathrm{Ab}$ for a genuine $G$-spectrum $E$. This will equip $\underline{\pi}_n(E)$ with the structure of a Mackey functor.

## 2 Wirthmüller Isomorphism

For a subgroup $H \subset G$, we can restrict the $G$-universe $\mathcal{U}$ to an $H$-universe, which we continue to denote by $\mathcal{U}$. Then we have the forgetful functor

$$\mathrm{Res}_H^G : G\mathscr{S}\mathcal{U} \to H\mathscr{S}\mathcal{U}$$

Its left adjoint is the **induced $G$-spectra** $G_+ \wedge_H -$ and its right adjoint is the **coinduced $G$-spectra** $F_H(G_+, -)$

Let $L(H)$ be the tangent $H$-representation of $eH$ in $G/H$. Note that it is $\{0\}$ when $G$ is a finite group.

**Theorem 2.1** (Wirthmüller Isomorphism)**.** *[EHCT, XVI.4.9] Let $H$ be a subgroup of $G$ and $X$ an $H$-spectrum. Then there is a natural weak equivalence of $G$-spectra*

$$F_H(G_+, \Sigma^{L(H)}X) \to G_+ \wedge_H X.$$

For a proof when $G$ is finite, see [Sch, Theorem 4.9] or [GHT, Theorem 3.2.15]..

**Consequence 1:** Now let $X$ in the above theorem be the sphere spectrum $\mathbb{S}$, we conclude that the orbits are self-dual when $G$ is finite, that is, the Spanier-Whitehead dual of $G/H$ is itself.

**Consequence 2:** We have

$$E_*^G(G_+ \wedge_H X) \cong E_*^H(\Sigma^{L(H)}X).$$

This compliments

$$E_G^*(G_+ \wedge_H X) \cong E_H^*(X).$$

## 3 Geometric Fixed points and Isotropy Separation Sequence

In this section, $G$ is a finite group. For $G$-spaces $X, Y$, we have $(X \wedge Y)^G \cong X^G \wedge Y^G$. This is not true for $G$-spectra. In fact, for suspension spectra, we have the Tom Dieck splitting theorem (will be in next week)

$$(\Sigma_G^\infty A)^G \simeq \bigvee_{(H)\subseteq G} \Sigma^\infty EWH_+ \wedge_{WH} A^H$$

where the index $(H) \subseteq G$ means the sum is running over all the conjugacy classes of subgroups of $G$ and $WH$ is the Weyl group $WH := N_G H/H$ of $H$. The geometric fixed point functor $\Phi^G : G\mathscr{S}\mathcal{U} \to \mathscr{S}\mathcal{U}^G$ is an alternative that enjoys the following nice properties

$$\Phi^G(\Sigma_G^\infty X) \simeq \Sigma^\infty X^G,$$

$$\Phi^G(X \wedge Y) \simeq \Phi^G X \wedge \Phi^G Y.$$

The conceptual way to defining $\Phi^G$ is as follows. Denote $\mathcal{P}$ the family of all proper subgroups of $G$. We can construct a $G$-CW complex $E\mathcal{P}$ which is universal in the sense that $E\mathcal{P}^H$ is contractible whenever $H$ is a proper subgroup and $E\mathcal{P}^G$ is empty. For example, this can be done by taking $\operatorname{colim}_n S(n\bar{\rho_G})$, where $S(n\bar{\rho_G})$ is the unit sphere in $n\bar{\rho_G}$.

The isotropy separation sequence is the cofiber sequence:

$$E\mathcal{P}_+ \to S^0 \to \widetilde{E\mathcal{P}}$$

where the first map is sending $E\mathcal{P}$ to the non-based point. By the above definition of $\widetilde{E\mathcal{P}}$, we see that it can be characterized by the universal property that $\widetilde{E\mathcal{P}}^H \simeq S^0$ for any proper subgroup $H$ and $\widetilde{E\mathcal{P}}^G \simeq *$.

Our model of the isotropy separation sequence is thus

$$S(\infty\bar{\rho_G})_+ \to S^0 \to S^{\infty\bar{\rho_G}}.$$

For a proper subgroup $H \subset G$, $\mathrm{Res}^G_H \rho_G = |G/H|\rho_H$. So we have $\bar{\rho_G}^H \neq 0$, which shows $(S^{\infty\bar{\rho_G}})^H \simeq S^\infty \simeq *$ and justifies our claim about the homotopy types.

**Definition 3.1.**

$$\Phi^G(X) = (X \wedge \widetilde{E\mathcal{P}})^G$$

The "geometric" in the name comes from the following equivalent definition. For simplicity, we do it for $\mathcal{U} = \infty\rho_G$. Here, $\rho_G = \mathbb{R}[G]$ the regular representation of the finite group $G$, so that $\mathcal{U}^G = \mathbb{R}^\infty$. The geometric-fixed-point-prespectra $\Phi^G D \in \mathscr{P}\mathcal{U}^G$ of a $G$-prespectrum $X$ is defined by:

$$\Phi^G X(V) := X(\rho_G \otimes V)^G \tag{1}$$

for $V \subset \mathscr{S}\mathcal{U}^G$, and with structure maps:

$$S^{W-V} \wedge \Phi^G X(V) \cong (S^{(W-V)\otimes\rho_G} \wedge X(\rho_G \otimes V))^G \xrightarrow{\sigma^G_{V,W}} X(\rho_G \otimes W)^G = \Phi^G X(W)$$

One then use the spectrification $L$ to turn it into a spectra. Although we have not explained the term $\Sigma$-cofibrant, we have a feeling of the equivalence of the two definitions from the following proposition:

**Proposition 3.2.** *[EHCT, XVI.3.4] For a $\Sigma$-cofibrant $G$-prespectra $D$, there is a weak equivalence $\Phi^G LD \simeq L\Phi^G D$.*

The geometric fixed points functor commutes with the suspension functor in the sense that the geometric fixed points $\Phi^G(\Sigma^\infty_G Y)$ is isomorphic to the suspension spectrum $\Sigma^\infty Y^G$ for any based $G$-space $Y$. This can be seen using the second construction above and the fact that the $G$-fixed points of the regular representation $\rho_G$ is $\mathbb{R}$. Namely, for any $V \in \mathscr{S}\mathcal{U}^G$,

$$\Phi^G(\Sigma^\infty_G Y)(V) = (Y \wedge S^{V\otimes\rho_G})^G \cong Y^G \wedge S^V = \Sigma^\infty Y^G(V).$$

We can define $\Phi^H : G\mathscr{S}\mathcal{U} \to (W_G H)\mathscr{S}\mathcal{U}^H$. First, we restrict $X \in G\mathscr{S}\mathcal{U}$ to $(N_G H)\mathscr{S}\mathcal{U}$, so that $H \subset N_G H$ is a normal subgroup. Then, for normal subgroups $N \subset G$, we define $\Phi^N : G\mathscr{S}\mathcal{U} \to (G/N)\mathscr{S}\mathcal{U}^N$ by taking the family $\mathscr{F}[N] = \{K \subset G | N \not\subset K\}$ and letting $\Phi^N(X) = (X \wedge \widetilde{E\mathscr{F}[N]})^N$. (Note that this $\mathscr{F}[N]$ is not the same as the previously used $\mathscr{F}(N)$.) It turns out that geometric fixed points also detect weak equivalence.

**Theorem 3.3.** *[EHCT, XVI.6.4] A map $f : E \to E'$ of $G$-spectra is an equivalence if and only if $\Phi^H f$ is a non-equivariant equivalence for all $H \subset G$.*

For a proof in the orthogonal spectra model, see [GHT, Theorem 7.12].

Let's wrap up the discussion of geometric fixed points by summarizing its properties, despite lacking the time to prove all of them:

**Remark 3.4.** The geometric fixed points functor $\Phi^G : G\mathscr{S}\mathcal{U} \to \mathscr{S}\mathcal{U}^G$ has the following properties

1. $\Phi^G$ is homotopy invariant which means it preserves $\pi_*$-isomorphism
2. $\Phi^G$ commutes with suspension, i.e. $\Phi^G \Sigma^\infty A^G \cong \Sigma^\infty A^G$ for any based $G$-CW complex $A$
3. $\Phi^G$ is symmetric monoidal
4. $\Phi^G$ commutes with filtered homotopy colimits

For completeness, we mention what happens in the model of orthogonal spectra. In orthogonal spectra, the naive fixed points is not "homotopically correct", i.e. it doesn't send $\pi_*$-isomorphisms of orthogonal $G$-spectra to $\pi_*$-isomorphisms of orthogonal spectra. However, the naive fixed points functor can be right derived via replacing a spectrum $X$ by a $\pi_*$-isomorphic $G$-$\Omega$-spectrum, and then take naive fixed points. This is denoted $F^G$. One can define the geometric fixed point functor $\Phi^G : \mathbf{Sp}^{\mathbf{O}}_{\mathbf{G}} \to \mathbf{Sp}^O$ using Equation 1; or defined it to be $F^G(\widetilde{E\mathcal{P}} \wedge X)$ as in [HHR]. These two definitions are equivalent:

**Proposition 3.5.** *[Sch, Proposition 7.6] For any orthogonal $G$-spectrum $X$, we have a map of spectra called evaluation map*

$$ev : F^G(\widetilde{E\mathcal{P}} \wedge X) \to \Phi^G X$$

*such that for any $G$-representation $W \in \mathcal{U}$, we have a weak equivalence $ev(W) : F^G(\tilde{E}\mathcal{P} \wedge X(W)) \to \Phi^G X(W)$.*

# 4 Homotopy Fixed points and Tate Construction

In this section, we'll see the homotopy fixed points, homotopy orbits and Tate constructions following [GM].

Recall that the fixed point functor $(-)^G$ in the category of $G$-spaces doesn't preserve weak equivalences. And we have seen that homotopy fixed points can actually detect (non-equivariant) weak equivalences of $G$-spaces. The homotopy fixed points of a $G$-space $A$ is given by the space of $G$-equivariant maps $\mathrm{Map}_G(EG_+, A)$ and the homotopy orbits is given by its Borel construction $EG_+ \wedge_G A$.

Now we define the stable analogue of homotopy fixed points and homotopy orbits.

**Definition 4.1.** The homotopy fixed points $X^{hG}$ of a $G$-spectrum $X$ is defined as the (derived) fixed point spectrum $F_G(EG_+, X)$ and the homotopy orbits $X_{hG}$ is defined as $EG_+ \wedge_G X$.

Denote $\widetilde{EG} := \mathrm{cofib}(EG_+ \to S^0)$. Since smashing with $X$ preserves cofiber sequence, we have another cofiber sequence:

$$EG_+ \wedge X \to X \to \widetilde{EG} \wedge X$$

And by the isomorphism $X \cong F(S^0, X)$, we get a map $X \cong F(S^0, X) \to F(EG_+, X)$. This induces a diagram:

$$\begin{array}{ccccc}
EG_+ \wedge X & \longrightarrow & X & \longrightarrow & \widetilde{EG} \wedge X \\
\downarrow{\scriptstyle \simeq} & & \downarrow & & \downarrow \\
EG_+ \wedge F(EG_+, X) & \longrightarrow & F(EG_+, X) & \longrightarrow & \widetilde{EG} \wedge F(EG_+, X)
\end{array}$$

It turns out that the left vertical map is always a $\pi_*$-isomorphism [GM, Proposition 1.2] (because it is a Borel equivalence $\wedge EG_+$) and $(EG_+ \wedge X)^G \simeq EG_+ \wedge_G X = X_{hG}$ as a special case of the Adams isomorphism, hence after taking the fixed points one has the following diagram

$$\begin{array}{ccccc}
X_{hG} & \longrightarrow & X^G & \longrightarrow & (\widetilde{EG} \wedge X)^G \\
\downarrow{\scriptstyle \simeq} & & \downarrow & & \downarrow \\
X_{hG} & \xrightarrow{\text{norm}} & X^{hG} & \longrightarrow & X^{tG}
\end{array}$$

where the right square is a pullback diagram and the lower right corner $X^{tG}$ is called the **Tate spectrum** of $X$.

Moreover, if we take $G$ to be a prime order cyclic group $C_p$, then $\widetilde{EG} \simeq \widetilde{E\mathcal{P}}$ and the top right corner becomes the geometric fixed point $\Phi^G X$ of $X$.

The name Tate construction comes from the following known fact: when we compute the homotopy groups $\pi_*(HM^{tG})$ of the Tate spectrum of the Eilenberg-Maclane spectrum of $M$, one recovers the Tate cohomology $\hat{H}^*(G; V)$ for a $G$-module $V = M(G/e)$.

Here is a short explanation of the Tate construction in the modern $\infty$-categorical language following [NS].

**Definition 4.2.** Let $\mathcal{C}$ be an $\infty$-category in which colimits and limits indexed over $BG$ exist. Define the homotopy orbits functor

$$\begin{aligned}(-)_{hG} : \mathcal{C}^{BG} &\to \mathcal{C} \\ F &\mapsto \mathrm{colim}_{BG} F\end{aligned}$$

and homotopy fixed points functor

$$\begin{aligned}(-)^{hG} : \mathcal{C}^{BG} &\to \mathcal{C} \\ F &\mapsto \lim_{BG} F\end{aligned}$$

Let $p : BG \to *$ be the canonical projection and $p^* : \mathcal{C} \to \mathcal{C}^{BG}$ be the pullback functor. Then $(-)_{hG}$ is left adjoint to $p^*$ and $(-)^{hG}$ is right adjoint to $p^*$. Let's put this into a more general context. Let $f : X \to Y$ be a map of Kan complexes, denote $f_!$ and $f_*$ the left adjoint and right adjoint functors of $p^*$, respectively. We will construct the norm map as a natural transformation $\mathrm{Nm}_f : f_! \to f_*$, So the norm map in our interest becomes a special case.

We still need to impose some conditions on $\mathcal{C}$. The condition we need is to assume $\mathcal{C}$ is a preadditive $\infty$-category, whose definition directly corresponds to the one in 1-category [HA, Definition 6.1.6.13]. We say that a map of $f : X \to Y$ of Kan complexes is $n$-truncated if all the homotopy fibers of $f$ has trivial homotopy groups at degree higher than $n$. Furthermore, we say a 1-truncated map is a *relative finite groupoid* if each fiber of $f$ has finitely many connected components and each of which is a classifying space of a finite group.

We refer the readers to [NS, Construction I.1.7] for the details of the construction of the norm transformation and summarize the result in the following proposition.

**Proposition 4.3.** *Let $\mathcal{C}$ be a preadditive $\infty$-category which has limits and colimits over all classifying spaces of finite groups. Let $f : X \to Y$ be a relative finite groupoid of Kan complexes, then both the left adjoint $f_!$ and right adjoint $f_*$ of $f^*$ exist, and there is a natural transformation :*

$$\mathrm{Nm}_f : f_! \to f_*$$

Now we can define the Tate construction in a stable $\infty$-category $\mathcal{C}$.

**Definition 4.4.** Let $\mathcal{C}$ be a stable $\infty$-category which admits all limits and colimits over $BG$. The Tate construction is the cofiber

$$\begin{aligned}(-)^{tG} : \mathcal{C}^{BG} &\to \mathcal{C} \\ X &\mapsto X^{tG} := \mathrm{cofib}(\mathrm{Nm}_G : X_{hG} \to X^{hG})\end{aligned}$$

For our interest, we consider the Tate construction in $\mathbf{Sp}$ the $\infty$-category of spectra.

**Example 4.5.** If we take the Eilenberg-Maclane spectrum $HM$ of the $G$-module $M$, then we recover the usual Tate cohomology via taking the homotopy groups of the Tate spectrum of $HM^{tG}$

$$\pi_*(HM^{tG}) \cong \hat{H}^{-*}(G, M)$$

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