Conference Manual

Shanghai Center for Mathematical Sciences

August 2019 · Shanghai

Contents

Appendix

Summer School on Equivariant Homotopy Theory

Shanghai Center for Mathematical Sciences & School of Mathematical Sciences

Fudan University, Shanghai, China

August 13-17, 2019

Scientific Committee

- Lars Hesselholt, Nagoya University & University of Copenhagen
- Peter May, University of Chicago

Lecturers

- Meng Guo, Perimeter Institute for Theoretical Physics & University of Toronto
- Hana Jia Kong, University of Chicago (Lead)
- Nicholas Kuhn, University of Virginia
- Ang Li, University of Kentucky
- Guchuan Li, Northwestern University
- Weinan Lin, University of Chicago
- Yunze Lu, University of Michigan
- Jonathan Rubin, University of California at Los Angeles
- XiaoLin Danny Shi, Harvard University
- Haoqing Wu, Swiss Federal Institute of Technology in Lausanne
- Mingcong Zeng, Utrecht University
- Ningchuan Zhang, University of Illinois at Urbana-Champaign
- Foling Zou, University of Chicago

Schedule for Summer School on Equivariant Homotopy Theory

Venue: Gu Lecture Hall, Room 220 of SCMS

Each lecture will have a 10-minute break in the middle. In the evening sessions, people will discuss problems from daytime lectures and there will also be short talks.

Tue. Aug. 13, Day 1: Nonequivariant background		
08:00-08:50	Registration (2nd Floor of SCMS)	
08:50-09:00	Opening Ceremony	
09:00-10:20	Yunze Lu	Lecture 1: Basic homotopy theory review
10:20-10:50	Coffee Break (2nd Floor of SCMS)	
10:50-12:10	Yunze Lu	Lecture 2: Cohomology theories and naive spectra
12:10-14:00	Lunch (Dining Hall 1st & 2nd Floor)	
14:00-15:20	Foling Zou	Lecture 3: Basics of spectra
15:50-17:10	Foling Zou	Lecture 4: Monoidal structures
17:10-19:00	Supper (Dining Hall 1st & 2nd Floor)	
19:00-	Evening session: Basic introduction to ∞-categories	
Wed. Aug. 14, Day 2: Equivariant homotopy theory		
09:00-10:20	Meng Guo	Lecture 5: Equivariant spaces
10:20-10:50	Coffee Break (2nd Floor of SCMS)	
10:50-12:10	Hana Jia Kong	Lecture 6: Presheaves on orbit category
12:10-14:00	Lunch (Dining Hall 1st & 2nd Floor)	
14:00-15:20	Weinan Lin	Lecture 7: Application
15:50-17:10	Ang Li	Lecture 8: Equivariant spheres, Freudenthal Suspension Theorem and the category of naive spectra
17:10-19:00	Supper (Dining Hall 1st & 2nd Floor)	
19:00-	Evening session: homology of symmetric groups, Barratt-Priddy theorem	
Thu. Aug. 15, Day 3: Equivariant stable homotopy theory		
09:00-10:20	Haoqing Wu	Lecture 9: Introduction to equivariant stable homotopy theory

10:20-10:50	Coffee Break (2nd Floor of SCMS)	
10:50-12:10	Jonathan Rubin	Lecture 10: Equivariant spectra and Mackey functors
12:10-14:00	Lunch (Dining Hall 1st & 2nd Floor)	
14:00-15:20	Haoqing Wu	Lecture 11: Orthogonal spectra
15:50-17:10	Weinan Lin	Lecture 12: Application
17:10-19:00	Supper (Dining Hall 1st & 2nd Floor)	
19:00-	Evening session: homological algebra of Mackey functors, equivariant cohomology of a point	
Fri. Aug. 16, Day 4: Advanced topics		
09:00-10:20	Ningchuan Zhang	Lecture 13: Equivariant K-theory
10:20-10:50	Coffee Break (2nd Floor of SCMS)	
10:50-12:10	Ningchuan Zhang	Lecture 14: Equivariant cobordism
12:10-14:00	Lunch (Dining Hall 1st & 2nd Floor)	
14:00-15:20	XiaoLin Danny Shi	Lecture 15: Introduction to HHR
15:50-17:10	XiaoLin Danny Shi	Lecture 16: The slice spectral sequence
17:10-18:00	Supper (Dining Hall 1st & 2nd Floor)	
Sat. Aug. 17, Day 5: Advanced topics (continued)		
09:00-10:20	Mingcong Zeng	Lecture 17: The Slice Differential Theorem, the Periodicity Theorem and the Homotopy Fixed Point Theorem
10:20-10:50	Coffee Break (2nd Floor of SCMS)	
10:50-12:10	Mingcong Zeng	Lecture 18: The Slice Differential Theorem, the Periodicity Theorem and the Homotopy Fixed Point Theorem (continued)
12:10-14:00	Lunch (Dining Hall 1st & 2nd Floor)	
14:00-15:00	Guchuan Li	Lecture 19: The Detection Theorem
15:20-16:20	Guchuan Li	Lecture 20: The Detection Theorem (continued)
16:40–17:40	Nicholas John Kuhn	Special Lecture: An introduction to the Balmer spectrum of G-equivariant finite spectra
17:40-18:30	Supper (Dining Hall 1st & 2nd Floor)	

Special Lecture

16:40–17:40, Aug. 17, Nicholas John Kuhn

Title: An introduction to the Balmer spectrum of G-equivariant finite spectra.

Abstract: Given a finite group G, Balmer and Sanders study the problem of identifying the tensor closed thick ideals of the homotopy category of finite G-spectra. Starting from the Hopkins-Smith nonequivariant Thick Subcategory Theorem, they identify the prime ideals using geometric fixed point functors. One then wants to know when one of these prime ideals is contained in another. By various reductions, to understand this, it suffices to solve the following problem: given n, and H<G, with G a finite p-group, how big can r be such that there exists a finite G-space with G-fixed points of chromatic type n, and H-fixed points of chromatic type n+r? In our talk, we will survey these results.

International Workshop on Algebraic Topology 2019

The International Workshop on Algebraic Topology is a three-day event which will take place at the Shanghai Center for Mathematical Sciences (SCMS) at Fudan University in Shanghai, China from August 19 to August 21, 2019. This is a joint event with the Summer School on Equivariant Homotopy Theory which will take place at the Center from August 13 to August 17, 2019.

The organizers gratefully acknowledge the support by Fudan University and National Natural Science Foundation of China.

Organizers

- Agnès Beaudry, University of Colorado Boulder
- Mark Behrens, University of Notre Dame
- Daniel Isaksen, Wayne State University
- Hana Jia Kong, University of Chicago
- Zhi Lü, Fudan University
- Guozhen Wang, Fudan University
- Zhouli Xu, Massachusetts Institute of Technology
- Yifei Zhu, Southern University of Science and Technology

- Assistant: Ying Zhao, Fudan University

Speakers

- Anna Marie Bohmann, Vanderbilt University
- Søren Galatius, University of Copenhagen
- David Gepner, University of Melbourne
- Jesper Grodal, University of Copenhagen
- Gijs Heuts, Utrecht University
- Nicholas Kuhn, University of Virginia
- Guchuan Li, Northwestern University
- Zhi Lü, Fudan University
- Lennart Meier, Utrecht University
- Mona Merling, University of Pennsylvania
- Jonathan Rubin, University of California at Los Angeles
- Nathalie Wahl, University of Copenhagen
- Zhouli Xu, Massachusetts Institute of Technology
- Min Yan, Hong Kong University of Science and Technology

Schedule for International Workshop on Algebraic Topology 2019

Venue: Gu Lecture Hall, Room 220 of SCMS

Mon. Aug. 19		
08:30-08:50	Registration (2nd Floor of SCMS)	
08:50-09:00	Opening Ceremony	
09:00-09:45	Jesper Grodal	String topology of finite groups of Lie type
09:45-10:05	Group Photo (1st Floor of SCMS)	
10:05–10:50	Zhi Lü	On orbit braids
10:50-11:10	Coffee Break (2nd Floor of SCMS)	
11:10–11:55	Anna Marie Bohmann	A multiplicative K-theory comparison
12:00-13:00	Lunch (Dining Hall 1st & 2nd Floor)	
14:30–15:15	Nathalie Wahl	The double of a simplicial complex
15:35–17:00	Graduate Student Talks	
18:00-20:00	Banquet (He Restaurant, 3rd Floor of U-Fun Shopping Mall 悠方购物公园瑞泰和江南菜)	
Tue. Aug. 20		
08:30-09:00	Coffee & Refreshments	
09:00-09:45	Nicholas John Kuhn	Looking backwards to move forwards: using the Smith construction to prove new results about the Balmer spectrum of equivariant stable homotopy
09:45-10:05	Coffee Break (2nd Floor of SCMS)	
10:05–10:50	Min Yan	Converse of Smith Theory
11:10–11:55	Jonathan Rubin	Characterizations of equivariant Steiner and linear isometries operads
12:00-13:00	Lunch (Dining Hall 1st & 2nd Floor)	
14:30–15:15	Zhouli Xu	The geography problem of 4-manifolds: 10/8 + 4
15:35–16:20	David Gepner	Orbispaces and elliptic cohomology
16:40–17:25	Mona Merling	G-manifolds and algebraic K-theory

17:30-18:15	Supper (Dining Hall 1st & 2nd Floor)	
Wed. Aug. 21		
08:30-09:00	Coffee & Refreshments	
09:00-09:45	Søren Galatius	Galois actions on symplectic K-theory
09:45-10:05	Coffee Break (2nd Floor of SCMS)	
10:05–10:50	Gijs Heuts	Koszul duality in v_n-periodic homotopy theory
11:10–11:55	Guchuan Li	Blue shift for real oriented cohomology theories
12:00-13:00	Lunch (Dining Hall 1st & 2nd Floor)	
14:30–15:15	Lennart Meier	Chromatic localizations of K-theory
15:35–16:40	Graduate Student Talks	
17:00-18:00	Supper (Dining Hall 1st & 2nd Floor)	

Wi-Fi Information

Wi-Fi: iFudan
ID: iwoat2019
Password: iwoat2019

Dining Arrangement

Meal vouchers for lunch and supper are provided which are accepted by the Dining Hall 1st & 2nd floor except the Halal Restaurant during this August. With this voucher, you can choose any one counter on the first or second floor of the Dining Hall and order up to four dishes with rice and a cup of yogurt.
Meal vouchers for breakfast are provided to participants staying at SCMS Guest House.

Please send the tableware to the designated reception after the meal.

Banquet
18:00, Aug.19
He Restaurant, 3rd Floor of U-Fun Shopping Mall 悠方购物公园瑞泰和江南菜

Reimbursement Information

To help with your reimbursement we kindly need you to prepare the following, put it into an envelope and hand it to Ying Zhao (Room 101 Shanghai Center for Mathematical Sciences).

1. Details of the purchase of tickets:

Up to the amount we have promised in the invitation, we can only reimburse the part of your airfare into and out of Shanghai. We need **the final receipt, e-ticket or invoice of your flight tickets.** If you travel multi-cities in China, please make sure that the reimbursable parts (prices in particular) are clearly singled out on these documents.

2. Boarding passes

Original copy for arrival trip. Scanned copy for departure trip and you can send it to Ying Zhao at yingzhao@fudan.edu.cn. If you buy the arrival and departure flight tickets separately, original copy for both arrival and departure trips are needed for reimbursement. Please get a scanned copy of the boarding passes before mailing it in case it got lost during the delivery.

3. Invoice and Details of Accommodation Fee

Up to the amount we have promised in the invitation, we can only reimburse part of accommodation fee for some participates. Both the invoice and the details of accommodation fee are needed.
The invoice **must** be with the header and tax number like the following
单位名称：复旦大学，纳税人识别号：12100000425006117P

4. Identity document & Bank Information

Please send the first page of Passport to Ying Zhao at yingzhao@fudan.edu.cn if you have not sent.
You can fill in the attached Form of information about your bank account or send the information to Ying Zhao at yingzhao@fudan.edu.cn. Please note that the bank account you provide has to be under your own name.
For Chinese citizen
● Bank Account Number

● Complete Subbranch Name in Chinese

For Non-Chinese citizen

● Beneficiary Name:

● Beneficiary's Address:

● Passport No.

● Bank Name:

● Bank Address:

● Beneficiary Account Number:

● SWIFT Code:

● Bank ABA (the US only):

● Bank IBAN (other countries):

Please note that the deadline to submit the materials for reimbursement is October 1st.

Address: Room 101 Shanghai Center for Mathematical Sciences, Fudan University Jiangwan Campus, No.2005 Songhu Road, Shanghai, China

Postcode: 200438

Addressee: Ying Zhao

E-mail: yingzhao@fudan.edu.cn

Tel: 021-31244000

Travel Information

Direction to SCMS:

From Pudong Airport (PVG) to SCMS:
1. Take a taxi (about 50 minutes, about 170 RMB).
2. Take Subway Line 2 East Extension Line to Guanglan Road Station, then change Subway Line 2 to Nanjing East Road Station, and then change Subway line 10 to Xin Jiang Wan Cheng Station, and walk for about 10 minutes. (About 20 RMB)
3. Take Maglev Speed Train to Long Yang Road Station (50 RMB), then take taxi (about 60 RMB).

From Hongqiao Airport (SHA) / Hongqiao Railway Station to SCMS:
1. Take a taxi (about 95 RMB).
2. Take Subway Line 10 to Xin Jiang Wan Cheng Station (5 RMB) and then walk about 10 minutes.

From Shanghai Railway Station to SCMS:
1. Take a taxi (about 40 RMB).
2. Take Subway Line 3, then Subway Line 8, and then Subway Line 10 to Xin Jiang Wan Cheng Station, and walk for about 10 minutes. (About 20 RMB)

For invited speakers, we have reserved a block of rooms free of charge at the SCMS Guest House.

SCMS Guest House
Address: No.2005 Songhu Road, Shanghai, China (Close to Gate No.3 of Fudan University Jiangwan Campus)
Tel: 021-31243777

In Chinese:
上海数学中心访问学者宿舍
地址：上海市杨浦区淞沪路 2005 号（复旦大学江湾校区 3 号门旁）
电话：021-31243777

For the other participants, we have reserved a block of rooms at the Green Hotel with dates of check-in August 12 and check-out August 22.

Green Hotel (Shanghai Fudan University Handan Road)
Address: 399 Handan Road, Yangpu District, Shanghai
Tel: 021-55885578

In Chinese:
格林豪泰智选酒店(上海复旦大学五角场地铁站店)
地址：上海市杨浦区邯郸路 399 号

We arrange pick-up service for participants staying at the Green Hotel.

Bus Schedule

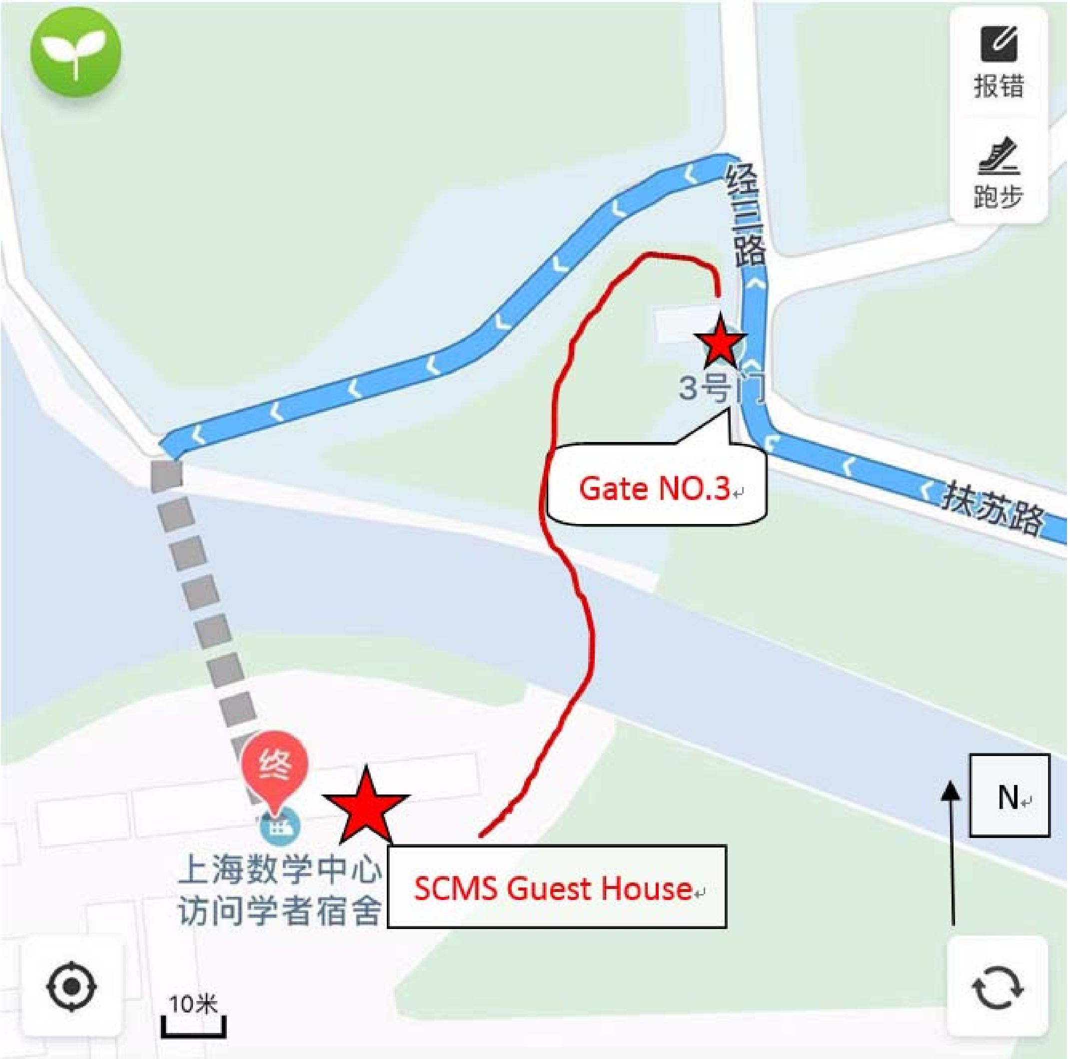
Date	Green Hotel to SCMS	SCMS to Green Hotel
Aug. 13	07:50	21:00
Aug. 14	08:30	18:00 21:00
Aug. 15	08:30	18:00 21:00
Aug. 16	08:30	18:00
Aug. 17	08:30	18:30
Aug. 19	08:10 14:00	12:50 20:00
Aug. 20	08:20 14:00	12:50 18:15
Aug. 21	08:20 14:00	12:50 18:00

You can also take Subway Line 10 from Wu Jiao Chang Station to Xin Jiang Wan Cheng Station or take Bus No. 168 from 五角场邯郸路站 to 复旦大学（新江湾城）站 and then walk to SCMS for the conference.

Campus Map

国帆路
复旦大学江湾新校区
生命科学学院
终
SCMS
国权北路
复旦大学
智华楼
Dining Hall
新江湾城
SMP滑板公园
新江湾城公园
综合保障楼
嘉誉湾
SCMS Guest House
加州水郡
U-Fun (悠方) shopping Mall
起
嘉誉国际广场
新江湾戎辉苑
Xinjiangwancheng Station
殷行路
新江湾城
新江湾戎德苑
上海音乐学院
实验学校
九龙仓玺园

Gate No.3 of Fudan University Jiangwan Campus → SCMS Guest House



Summer School on Equivariant Homotopy Theory
Shanghai Center for Mathematical Sciences
August 13–17, 2019

Updated August 8, 2019

Scientific Committee:
Lars Hesselholt, Nagoya University & University of Copenhagen
Peter May, University of Chicago

Organizers:
Agns Beaudry, University of Colorado Boulder
Mark Behrens, University of Notre Dame
Daniel Isaksen, Wayne State University
Hana Jia Kong, University of Chicago
Zhi L, Fudan University
Guozhen Wang, Fudan University
Zhouli Xu, Massachusetts Institute of Technology
Yifei Zhu, Southern University of Science and Technology

1 Day 1: Nonequivariant background

We review the basics of homotopy theory, with the Homotopy Hypothesis in mind. We introduce the notion of infinity categories, and make the point that we should use homotopy limits and colimits in infinity categories. Finally we introduce the universal properties of presheaves in infinity categories, and give the slogan that many of the categories we use, such as spectra and symmetric monoids, are (localizations of) infinity presheaves.

1.1 Talk 1: basic homotopy theory review

1. fiber and cofiber sequences: universal properties, homotopy limits and colimits in general. ([16] 8.4, 8.6, [11] 5.2.2.7)
2. CW complexes: Top is generated by one point under colimits. ([16] 10.1, [11] 5.1.5.8)
3. homotopy categories: CW approximation, weak equivalence, Puppe sequence. intuitions from ∞-categories. ([16] 10.4, [11] 1.1)
4. Postnikov towers: truncations as right adjoints. ([11] 1.1.1.4)

1.2 Talk 2: cohomology theories and naive spectra

1. axiomatic cohomology: universal properties of the category Top. ([16] 18.1, [11] 5.1.5.6)
2. ordinary cohomology as functors to the derived category, review derived category. ([6] 4.1, [12] 1.3.3)
3. Eilenberg-Maclane spaces, Omega-spectra. ([16] 16.5)
4. Brown representability theorem, represented cohomology theories, representability as the existence of adjoints. ([11] 5.5.2.9)

1.3 Talk 3: basics of spectra

1. Need for good homotopy category of spectra, some history. ([2] III.1, III.2)
2. stable ∞-categories and universal properties of stabilization. ([12] 1.1, 1.4)
3. examples of spectra: K-theory, cobordism, etc. ([2] III.11, I.2)
4. duality: Spanier-Whitehead, Atiyah, Grothendieck, etc. ([2] III.5, III.10)

1.4 Talk 4: monoidal structures

1. monoidal categories: monoids in the symmetric monoidal category of categories. ([12] 2.4.2 (a))
2. ∞-category settings, E_∞-objects. ([12] 2.0.0.7, [14] 3.5)
3. monoidal structure on the category of spectra: smash products, monoidal properties of stabilization. ([2] III.9, [12] 7.1)
4. Day convolution and monoidal structures on presheaves. ([13] 21.6)
5. orthogonal spectra: definition, point of view as presheaves. ([21] 3.1.3, [13] 4.4)

1.5 Evening

simplicial sets, Kan complexes, nerves, quasi-categories. ([11] 1.1, [7] Chapter 1)

2 Day 2: Equivariant homotopy theory

We give basics of equivariant homotopy theory. We introduce Elmendorf's theorem to give the idea that naive equivariant spaces are infinity presheaves over BG and genuine equivariant spaces are presheaves over the orbit category, and use the universal properties of presheaves to understand the axioms of equivariant cohomology theories. The last talk introduces the notion of suspension by a representation sphere, as the preliminaries for equivariant stable homotopy.

2.1 Talk 1: equivariant spaces

1. group actions, G-spaces. ([15] I.1)
2. G-CW complexes: G-spaces are generated by G/H's under colimits. ([15] I.3, [11] 5.1.6.11)
3. fixed point and orbit adjunction. ([15] I.1)
4. homotopy orbits and homotopy fixed points: universal properties, as left/right adjoints of forgetful functor. ([15] V.2, [11] 1.2.13)
5. homotopy groups and the Whitehead theorem, weak equivalence of equivariant spaces. ([15] I.3)

2.2 Talk 2: presheaves on orbit category

1. orbit category: equivariant spaces as presheaves in spaces, the equivalence of homotopy categories. ([15] V.3, [11] 5.1.6.11)
2. universal properties of presheaves. ([11] 5.1.5.6)
3. examples: equivariant cohomology with values in a derived category using universal properties of presheaves. ([15] I.4)

2.3 Talk 3: application

1. ordinary equivariant homology: coefficient systems, axiomatic and Bredon cohomology of G-spaces. ([15] I.4, I.6)
2. Smith theory. ([15] IV.1)

2.4 Talk 4: equivariant spheres

1. orthogonal representations of finite groups, universes of G-representations. ([15] IX.1, IX.2)
2. G-spheres and $RO(G)$-graded homotopy groups. ([15] IX.5)
3. the equivariant suspension theorem, stable homotopy groups. ([15] IX.4, X.6)
4. universal properties of equivariant stabilization, naive equivariant spectra. ([20] 2.1)

2.5 Evening

homology of symmetric groups, plus construction, Barratt-Priddy theorem. ([4] I.5)

3 Day 3: Equivariant stable homotopy theory

We give two points of view for equivariant spectra. First we view them as stabilization of equivariant spaces by all the representation spheres, and give the classical construction using universes, and also Robalo's construction using infinity category techniques. Second we view equivariant spectra as stable infinity sheaves over the enriched Burnside category. The latter makes the introduction of Mackey functors very natural. Finally we introduce orthogonal spectra, and those definitions and theorems using orthogonal spectra.

3.1 Talk 1: introduction to equivariant stable homotopy theory

1. G-prespectra and G-spectra in a universe, stable homotopy groups of G-spectra. ([15] XII.2)
2. Wirthmuller isomorphism, transfers. ([21] 3.2)
3. homotopy fixed points, homotopy orbits: using universal properties, Tate spectra. ([11] 1.2.13, [17] 1.1, [15] XXI.1)
4. fixed points, geometric fixed points: defined using a universe. ([21] 3.3)
5. isotropy separation sequence, characterization of equivariant equivalences via geometric fixed points. ([21] 3.3)

3.2 Talk 2: equivariant spectra as presheaves

1. Burnside category, the isomorphism between the equivariant 0-stem and the Burnside ring. ([15] XVII.2)
2. basics of Mackey functors, the monoidal structure on Mackey functors. Mackey functors as presheaves over Burnside category. ([15] IX.4)
3. presheaves over the Burnside category with values in spectra. ([9] 0.1, [12] 1.4.4.9)

3.3 Talk 3: orthogonal spectra

1. definitions: equivariant setting, global equivariant spectra. ([21] 3.1)
2. suspension spectra, geometric fixed points of suspension spectra. ([21] 3.3)
3. smash products. ([21] 3.5)
4. the tom Dieck splitting (original formulation). ([22] 6.12)
5. the Adams isomorphism; the combination of tom Dieck splitting and Adams isomorphism. ([1], [19])
6. norms (for commutative orthogonal G-ring spectra), the HHR norm functor. ([21] 5.1)

3.4 Talk 4: application

1. equivariant Eilenberg-Maclane spectra: $RO(G)$-graded cohomology, existence of Bredon cohomology, Brown representability, the existence of HM. ([15] XIII.4, XIII.3)
2. Conner conjecture. ([15] 9.6, [18])

3.5 Evening

homological algebra of Mackey functors, equivariant cohomology of a point.

4 Day 4: Advanced topics

We give many examples in equivariant stable homotopy theory. The construction of $MU^{((G))}$ will be treated in detail as the preliminaries for [10].

4.1 Talk 1: equivariant K-theory

1. Borel equivariant theories from non-equivariant spectra. ([21] 4.5.21)
2. connective and periodic equivariant K-theory spectra, comparison with equivariant vector bundles, Greenlees's "equivariant connective K-theory". ([21] 6.3, 6.4, [8])
3. real K-theory. ([3])
4. Atiyah-Segal completion theorem. comparison of fixed points and homotopy fixed points. ([15] XIV.5)

4.2 Talk 2: equivariant cobordism

1. different equivariant flavors of Thom spectra (connective versus orientable). ([21] 6.1)
2. equivariant Thom-Pontryagin constructing and when it is an isomorphism. ([21] 6.2)
3. real cobordism. ([10] 5.2)
4. the construction of $MU^{((G))}$. ([10] 5.1)

4.3 Talk 3: introduction of HHR

([10] Section 1)

1. historical introduction and overview.
2. the main steps in HHR.

4.4 Talk 4: the slice spectral sequence

1. the slices and the slice filtration. ([10] Section 4)
2. Dugger's computation of KR-theory. ([5])
3. the slices of $MU^{((G))}$: the slice theorem, the reduction theorem, and the gap theorem. ([10] Section 6)

5 Day 5: HHR on Kervaire invariant one (up to the experts)

This day is devoted to [10], giving the proofs of the main steps for the Kervaire invariant one problem.

5.1 Talk 1: the slice differential theorem and the periodicity theorem

([10] Section 6, Section 9)

5.2 Talk 2: the homotopy fixed points theorem

([10] Section 10)

5.3 Talk 3: the detection theorem preliminaries

Adams-Novikov, formal A-modules, etc. ([2] III.15, [10] 11.2)

5.4 Talk 4: the detection theorem

proof of the detection theorem. ([10] Section 11)

References

[1] J. F. Adams, *Stable homotopy and generalised homology*, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, IL, 1995, Reprint of the 1974 original. MR1324104

[2] J. F. Adams, *Prerequisites (on equivariant stable homotopy) for Carlsson's lecture*, Algebraic topology, Aarhus 1982 (Aarhus, 1982), Lecture Notes in Math., vol. 1051, Springer, Berlin, 1984, pp. 483–532. MR764596

[3] M. F. Atiyah, *K-theory and reality*, Quart. J. Math. Oxford Ser. (2) **17** (1966), 367–386. MR0206940

[4] F. R. Cohen, T. J. Lada, and J. P. May, *The homology of iterated loop spaces*, Lecture Notes in Mathematics, Vol. 533, Springer-Verlag, Berlin-New York, 1976. MR0436146

[5] D. Dugger, *An Atiyah-Hirzebruch spectral sequence for KR-theory*, K-Theory **35** (2005), no. 3-4, 213–256 (2006). MR2240234

[6] S. I. Gelfand and Yu. I. Manin, *Homological algebra*, Springer-Verlag, Berlin, 1999, Translated from the 1989 Russian original by the authors, Reprint of the original English edition from the series Encyclopaedia of Mathematical Sciences [ıt Algebra, V, Encyclopaedia Math. Sci., 38, Springer, Berlin, 1994; MR1309679 (95g:18007)]. MR1698374

[7] P. G. Goerss and J. F. Jardine, *Simplicial homotopy theory*, Modern Birkhäuser Classics, Birkhäuser Verlag, Basel, 2009, Reprint of the 1999 edition [MR1711612]. MR2840650

[8] J. P. C. Greenlees, *Equivariant connective K-theory for compact Lie groups*, J. Pure Appl. Algebra **187** (2004), no. 1-3, 129–152. MR2027899

[9] B. Guillou, J. P. May, *Models of G-spectra as presheaves of spectra.* arXiv:1110.3571

[10] M. A. Hill, M. J. Hopkins, and D. C. Ravenel, *On the nonexistence of elements of Kervaire invariant one*, Ann. of Math. (2) **184** (2016), no. 1, 1–262. MR3505179

[11] J. Lurie, *Higher topos theory*, Annals of Mathematics Studies, vol. 170, Princeton University Press, Princeton, NJ, 2009. MR2522659

[12] J. Lurie, *Higher Algebra*, available at `http://www.math.harvard.edu/~lurie/papers/HA.pdf`.

[13] M. A. Mandell, J. P. May, S. Schwede, and B. Shipley, *Model categories of diagram spectra*, Proc. London Math. Soc. (3) **82** (2001), no. 2, 441–512. MR1806878

[14] J. P. May, *The geometry of iterated loop spaces*, Springer-Verlag, Berlin-New York, 1972, Lectures Notes in Mathematics, Vol. 271. MR0420610

[15] J. P. May, *Equivariant homotopy and cohomology theory*, CBMS Regional Conference Series in Mathematics, vol. 91, Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 1996, With contributions by M. Cole, G. Comezaña, S. Costenoble, A. D. Elmendorf, J. P. C. Greenlees, L. G. Lewis, Jr., R. J. Piacenza, G. Triantafillou, and S. Waner. MR1413302

[16] J. P. May, *A concise course in algebraic topology*, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, IL, 1999. MR1702278

[17] T. Nikolaus and P. Scholze, *On topological cyclic homology*, Acta Math. **221** (2018), no. 2, 203–409. MR3904731

[18] R. Oliver, *A proof of the Conner conjecture*, Ann. of Math. (2) **103** (1976), no. 3, 637–644. MR0415650

[19] H. Reich and M. Varisco, *On the Adams isomorphism for equivariant orthogonal spectra*, Algebr. Geom. Topol. **16** (2016), no. 3, 1493–1566. MR3523048

[20] M. Robalo, *K-theory and the bridge from motives to noncommutative motives*, Adv. Math. **269** (2015), 399–550. MR3281141

[21] S. Schwede, *Global homotopy theory*, New Mathematical Monographs, vol. 34, Cambridge University Press, Cambridge, 2018. MR3838307

[22] S. Schwede, *Lectures on equivariant stable homotopy theory*, available at http://www.math.uni-bonn.de/people/schwede/equivariant.pdf.

International Workshop on Algebraic Topology 2019

Schedule and Abstracts

All talks take place in the Gu Lecture Hall, Room 220 of the Shanghai Center for Mathematical Sciences.

8/19, Monday

08:30 - 09:00, Coffee & Refreshments

09:00 - 09:45, Jesper Grodal, String topology of finite groups of Lie type

Abstract:
Finite groups of Lie type, such as $\mathrm{SL}_n(\mathbb{F}_q)$, $\mathrm{Sp}_n(\mathbb{F}_q)$ $\ldots$, are ubiquitous in mathematics, and calculating their cohomology has been a central theme over the years. Without any structural reasons as to why, it has calculationally been observed that, when calculable, their mod ℓ cohomology agree with the mod ℓ cohomology of $LBG(\mathbb{C})$, the free loop space on $BG(\mathbb{C})$, the classifying space of the corresponding complex algebraic group $G(\mathbb{C})$, as long as q is congruent to 1 mod ℓ. This is despite that $LBG(\mathbb{C})$ and $BG(\mathbb{F}_q)$ are vastly different spaces, also at a prime ℓ, ruling out some space-level equivalence. In recent joint work with Anssi Lahtinen, that combines ℓ-compact groups with string topology à la Chas-Sullivan, we give a general structural relationship between these two cohomologies, which, suitably formulated, even works without any congruence condition on q, as long as it is prime to ℓ. We use this to prove structured versions of previous calculations, and establish isomorphism in new cases. The isomorphism conjecture in general hinges on the fate of a single cohomology class in exceptional Lie groups at small primes. My talk will begin to tell this story, as we know it so far $\ldots$

10:05 - 10:50, Zhi Lü, On orbit braids

Abstract:
Let M be a connected topological manifold of dimension at least 2 with an effective action of a finite group G. Associating with the orbit configuration space $F_G(M,n), n \geq 2$ of the G-manifold M, we try to upbuild the theoretical framework of orbit braids in $M \times I$ where the action of G on I is trivial, which contains the following contents: we introduce the orbit braid group $\mathcal{B}_n^{orb}(M,G)$, and show that it is isomorphic to a group with an additional endowed operation (called the extended fundamental group), formed by the homotopy classes of some paths (not necessarily closed paths) in $F_G(M,n)$, which is an essential extension for fundamental groups. The orbit braid group $\mathcal{B}_n^{orb}(M,G)$ is large enough to contain the fundamental group of $F_G(M,n)$ and other various braid groups as its subgroups. Around the central position of $\mathcal{B}_n^{orb}(M,G)$, we obtain five short exact sequences weaved in a commutative diagram. We also analyze the essential relations among various braid groups associated to those configuration spaces $F_G(M,n), F(M,n)$, and $F(M/G,n)$. We finally consider how to give the presentations of orbit braid groups in terms of orbit braids as generators. We carry out our work by choosing $M = \mathbb{C} \approx \mathbb{R}^2$ with typical actions of $\mathbb{Z}_p$ and $(\mathbb{Z}_2)^2$. We obtain the presentations of the corresponding orbit braid groups, from which we see that the generalized braid group $Br(D_n)$ (introduced by Brieskorn) actually agrees with the orbit braid group $\mathcal{B}_n^{orb}(\mathbb{C} \setminus \{0\}, \mathbb{Z}_2)$ and $Br(D_n)$ is a sub-

group of the orbit braid group $\mathcal{B}_n^{orb}(\mathbb{C}, \mathbb{Z}_2)$. This talk is based upon a joint work with Hao Li and Fengling Li.

11:10 - 11:55, Anna Marie Bohmann, A multiplicative K-theory comparison

Abstract:
K-theory is a central object of study that relates algebraic topology, number theory and geometric topology, among other fields. There are many ways of constructing K-theory. Classical results tell us that these are all "the same" in an additive sense. In this talk, I will discuss joint work with Osorno in which we prove a multiplicative comparison of two classic K-theory constructions, those of Segal and Waldhausen. In particular, this produces comparisons of commutative ring spectra and spectrally enriched categories.

12:00, Lunch Break

02:30 - 03:15, Nathalie Wahl, The double of a simplicial complex

Abstract:
We define the double of a simplicial complex and study its connectivity properties. The definition is motivated by homological stability: one can study homological stability for a sequence of groups $G_1 \to G_2 \to G_3 \to \dots$ using an associated simplicial complex; the double of this complex is related to stabilizing in steps of two $G_2 \to G_4 \to G_6 \to \dots$. Because of this, homological stability suggests that if a simplicial complex is highly connected, so is its double, a result that we prove under appropriate assumptions. (This is joint work with Kathryn Lesh and Bridget Schreiner.)

03:35 - 05:00, Graduate Student Talks

06:00, Banquet

8/20, Tuesday

08:30 - 09:00, Coffee & Refreshments

09:00 - 09:45, Nick Kuhn, Looking backwards to move forwards: using the Smith construction to prove new results about the Balmer spectrum of equivariant stable homotopy

Abstract:
Balmer and Sanders show that understanding the problem of determining inclusions among the tensor triangulated thick ideals of finite G-spectra can be reduced to the following problem: given n, and $H < G$, with G a finite p-group, how big can r be such that there exists a finite G-complex with G-fixed points of chromatic type n, and H-fixed points of chromatic type $n + r$? Barthel et al. solve this problem when G is abelian by showing that a "blue shift" upper bound that they establish is realized with a family of examples previously analyzed by Arone and Lesh, and basically due to Mitchell.

In my talk I will discuss recent and ongoing joint work with my student Chris Lloyd, on a way to construct examples that is both much more elementary and much more flexible than previous constructions. The idea is to feed lens spaces associated to representations of G into Jeff Smith's "machine" for constructing type n complexes: the examples are thus well chosen stable wedge summands of smash products of lens spaces associated to well chosen G-representations. The method yields alternatives to the Arone–Lesh examples and new examples too, and is well suited for investigation by computer. For example, we are able to resolve the first unknown case, when G is the dihedral group of order 8 and H is a noncentral subgroup of order 2:

the maximal shift is 2, not 1, as was previously thought.

10:05 - 10:50, Min Yan, Converse of Smith Theory

Abstract:
Suppose G is a finite group, and f is a map from a CW complex F to the fixed point of a G-CW complex Y. Is it possible to extend F to a finite G-CW complex X satisfying $X^G = F$, and extend f to a G-map $g\colon X \to Y$, such that g is a homotopy equivalence after forgetting the G-action?

In case Y is a single point, the problem becomes whether a given finite CW complex F is the fixed point of a G-action on a finite contractible CW complex. In 1942, P.A. Smith showed that the fixed point of a p-group action on a finite $\mathbb{Z}_p$-acyclic complex is still $\mathbb{Z}_p$-acyclic. In 1971, Lowell Jones proved a converse for semi-free cyclic group action on finite contractible X. In 1975, Robert Oliver proved that, for general action on finite contractible X, if the order of G is not prime power, then the only obstruction is the Euler characteristic of F.

We extend the classical results of Lowell Jones and Robert Oliver to the general setting. For semi-free action, we encounter a finiteness type obstruction. For general action by group of not prime power order, the obstruction is the Euler characteristics over components of Y^G. We calculate such obstructions for various examples.

This is a joint work with Sylvain Cappell of New York University, and Shmuel Weinberger of University of Chicago.

11:10 - 11:55, Jonathan Rubin, Characterizations of equivariant Steiner and linear isometries operads

Abstract:
Representation spheres and universes are prominent in classical equivariant stable homotopy theory, but more recent approaches to the subject emphasize algebraic structure over the representation theory. In this talk, I will discuss how these two perspectives are reflected on the level of N_∞ operads.

Roughly speaking, a N_∞ operad is a structure that parametrizes homotopy commutative monoids equipped with additional equivariant transfer maps. Since Blumberg and Hill's groundbreaking work, it has been known that the homotopy theory of N_∞ operads is essentially algebraic. However, this viewpoint ignores the peculiarities of the natural geometric examples, namely the Steiner and linear isometries operads over incomplete universes. I will describe a few cases where one can characterize such operads in purely algebraic terms, and I will explain why for most groups, there are N_∞ operads that are not equivalent to any Steiner or linear isometries operad.

12:00, Lunch Break

02:30 - 03:15, Zhouli Xu, The geography problem of 4-manifolds: $10/8 + 4$

Abstract:
A fundamental problem in 4-dimensional topology is the following geography question: which simply connected topological 4-manifolds admit a smooth structure? After the celebrated work of Kirby–Siebenmann, Freedman, and Donaldson, the last uncharted territory of this geography question is the "11/8-Conjecture." This conjecture, proposed by Matsumoto, states that for any smooth spin 4-manifold, the ratio of its second Betti number and signature is least 11/8.

Furuta proved the "$10/8 + 2$"-Theorem by studying the existence of certain Pin(2)-equivariant stable maps between representation spheres. In this talk, we will present a complete solution to Furuta's problem

by analyzing the Pin(2)-equivariant Mahowald invariants. In particular, we improve Furuta's result into a "10/8+4"-Theorem. Furthermore, we show that within the current existing framework, this is the limit. This is a joint work with Mike Hopkins, Jianfeng Lin and XiaoLin Danny Shi.

03:35 - 04:20, David Gepner, Orbispaces and elliptic cohomology

Abstract:
The homotopy theory of orbispaces is a generalization of the homotopy theory of G-spaces in which the group G is allowed to vary. More precisely, orbispaces model the homotopy theory of topological Artin stacks (a.k.a. unstable global homotopy theory) in much the same way as Kan complexes model the homotopy theory of topological spaces. We will show how certain geometrically defined cohomology theories, such as topological K-theory and elliptic cohomology, extend to orbispaces, and how this extension is useful for both theoretical and computational purposes. Finally, we will discuss some calculations in equivariant elliptic cohomology.

04:40 - 05:25, Mona Merling, G-manifolds and algebraic K-theory

Abstract:
Waldhausen's algebraic K-theory of spaces provides a critical link in the classification of manifolds and their diffeomorphisms. We will explain this connection and discuss work in progress with Cary Malkiewich on the analogous story for manifolds with group action.

8/21, Wednesday

08:30 - 09:00, Coffee & Refreshments

09:00 - 09:45, Søren Galatius, Galois actions on symplectic K-theory

Abstract:
The symplectic K-theory groups $K\mathrm{Sp}_*(\mathbb{Z})$ may be defined similarly to usual algebraic K-theory $K_*(\mathbb{Z})$, replacing the group $\mathrm{GL}_n(\mathbb{Z})$ by the symplectic group $\mathrm{Sp}_{2g}(\mathbb{Z})$. Much is known about the relationship between $K\mathrm{Sp}_*(\mathbb{Z})$ and $K_*(\mathbb{Z})$, due to work of Karoubi and others. I will explain that large cyclic subgroups of $\mathrm{Sp}_{2g}(\mathbb{Z})$ may be used to detect an important part of $K\mathrm{Sp}_*(\mathbb{Z})$, and furthermore to understand an action of field automorphisms of the complex numbers on the p-adic completion. This is joint work with T. Feng and A. Venkatesh.

10:05 - 10:50, Gijs Heuts, Koszul duality in v_n-periodic homotopy theory

Abstract:
Following the rational homotopy theory of Quillen and Sullivan, one can compute the rational homotopy groups of a space from a commutative algebra model of its cochains by taking (derived) indecomposables. More abstractly, this procedure implements a form of Koszul duality between commutative algebras and Lie algebras. The Lie algebra model of rational homotopy theory generalizes to v_n-periodic homotopy theory; however, the above procedure generalizes only partially. Work of Behrens–Rezk shows that the v_n-periodic homotopy groups of spheres and of certain compact Lie groups can be computed from their cochains valued in Morava E-theory (or some variant thereof), but in general it is unclear for which spaces this works. I will report on ongoing joint work with Brantner, Hahn, and Yuan, which provides a large class of spaces for which "Koszul duality works" also in v_n-periodic homotopy.

11:10 - 11:55, Guchuan Li, Blue shift for real oriented cohomology theories

Abstract:
This is joint work with Vitaly Lorman and James D. Quigley. The $\mathbb{Z}/p$-Tate cohomology spectrum of the n'th Johnson–Wilson theory splits as a wedge of $(n-1)$'th Johnson–Wilson theories (after completion). We construct a C_2-equivariant lifting of this splitting for Real Johnson–Wilson theories. The C_2-fixed points of this splitting is a higher height analogue to Davis and Mahowald's splitting of the Tate cohomology spectrum of ko as a wedge of $H\mathbb{Z}$.

12:00, Lunch Break

02:30 - 03:15, Lennart Meier, Chromatic localizations of K-theory

Abstract:
Red shift is the philosophy that the chromatic height n part of a ring spectrum controls the chromatic height $(n+1)$ part of its algebraic K-theory. A recent result of Bhatt, Clausen and Mathew shows that this philosophy is precisely true if the ring spectrum is a discrete ring. I will report on joint work with Markus Land and Georg Tamme, where we find a new proof of this result and provide generalizations to ko- and tmf-algebras.

03:35 - 04:40, Graduate Student Talks

To help with your reimbursement we kindly need you to prepare the following, put it into an envelope and hand it to Ying Zhao (Room 101 Shanghai Center for Mathematical Sciences).

1. Details of the purchase of tickets:

Up to the amount we have promised in the invitation, we can only reimburse the part of your airfare into and out of Shanghai. We need **the final receipt, e-ticket or invoice of your flight tickets.** If you travel multi-cities in China, please make sure that the reimbursable parts (prices in particular) are clearly singled out on these documents.

2. Boarding passes

Original copy for arrival trip. Scanned copy for departure trip and you can send it to Ying Zhao at yingzhao@fudan.edu.cn. If you buy the arrival and departure flight tickets separately, original copy for both arrival and departure trips are needed for reimbursement. Please get a scanned copy of the boarding passes before mailing it in case it got lost during the delivery.

3. Invoice and Details of Accommodation Fee

Up to the amount we have promised in the invitation, we can only reimburse part of accommodation fee for some participates. Both the invoice and the details of accommodation fee are needed.

The invoice **must** be with the header and tax number like the following

单位名称：复旦大学，纳税人识别号：**12100000425006117P**

4. Identity document & Bank Information

Please send the first page of Passport to Ying Zhao at yingzhao@fudan.edu.cn if you have not sent.

You can fill in the attached Form of information about your bank account or send the information to Ying Zhao at yingzhao@fudan.edu.cn. Please note that the bank account you provide has to be under your own name.

Please note that the deadline to submit the materials for reimbursement is October 1st.

<u>For your reimbursement:</u>

FORM 1 For Chinese citizen（Please fill in the form in Chinese.）

Beneficiary’s Name:	
Bank Account Number	
Complete Subbranch Name	

FORM 2 For Non-Chinese citizen

Beneficiary’s Name:	
Beneficiary’s Address:	
Passport No.	
Bank Name:	
Bank Address:	
Beneficiary Account Number:	
SWIFT Code:	
Bank ABA (the US only):	
Bank IBAN (other countries):	