

**IWOAT SUMMER SCHOOL 2024:
MOTIVIC STABLE HOMOTOPY THEORY**

Day 1: 4 Lectures: Introduction to motivic categories.

Lecture 1-1 Tom Bachmann

Overview.

Lecture 1-2 Thomas Brazelton

Definition of infinity-categories as quasi-categories, the existence of mapping spaces, interpretation. Presentable infinity categories. Bousfield localization. All of this will have to be presented very tersely, mostly as a reminder.

Lecture 1-3 Thomas Brazelton

The construction of the unstable \mathbb{A}^1 -homotopy category over a base. Functorialities, f_* , f^* . Thom spaces. Homotopy purity.

Lecture 1-4 Tom Bachmann

The stable \mathbb{A}^1 -homotopy category. As \mathbb{P}^1 -spectra. Suspension functors, “ambidexterity”, purity. 6 functors (just construction and properties).

Day 2: 4 Lectures: connectivity, homotopy t -structure and Morel’s theorem on stable π_0 .

Lecture 2-1 Viktor Burghardt

Morel’s S^1 - \mathbb{A}^1 -connectivity theorem. Introduce S^1 -spectra, follow Morel’s paper “The stable \mathbb{A}^1 -connectivity theorems”.

Lecture 2-2 Viktor Burghardt

Promote the connectivity theorem for \mathbb{P}^1 -spectra. Introduce the S^1 and \mathbb{P}^1 homotopy t -structures.

Lecture 2-3 Fangzhou Jin

The heart of the homotopy t -structure as homotopy modules, following Morel’s ICTP notes.

Lecture 2-4 Fangzhou Jin

Introduce the Milnor-Witt K -sheaves. State the main properties without much proof, following the ICTP notes, state Morel’s theorems on stable $\pi_{n,n}$, and give a sketch of the proof, following the ICTP notes.

Day 3: 2 Lectures. Motivic cohomology.

Lecture 3-1 Brian Shin

Motivic cohomology via the Voevodsky motivic complexes

Lecture 3-2 Brian Shin

Representing motivic cohomology in $\mathrm{SH}(k)$, following Røndigs-Østvær.

Day 4. 4 Lectures. K -theory, algebraic cobordism, and the slice tower.

Lecture 4-1 Longke Tang

BGL and the unstable representation theorem of Morel-Voevodsky.

Lecture 4-2 Nikolai Opdan

KGL and the stable representation theorem.

Lecture 4-3 Niklas Kipp

The construction of MGL. The formal group law. Computation of motivic cohomology of MGL.

Lecture 4-4 Niklas Kipp

Introduction to Voevodsky's slice tower.

Day 5. 4 Lectures. Hopkins-Morel-Hoyois theorem, slices of KGL and MGL.

Lecture 5-1 Klaus Mattis

Statement of the HMH theorem: Introduce the Steenrod algebra and the motivic version as a black box.

Lecture 5-2 Longke Tang

Computation of the homology (rational and mod ℓ) of regular quotients of MGL.

Lecture 5-3 Klaus Mattis

Finish the proof of the HMH theorem.

Lecture 5-4 Nikolai Opdan

Compute the slices of KGL and MGL.