IWOAT SUMMER SCHOOL 2024:
MOTIVIC STABLE HOMOTOPY THEORY

Day 1: 4 Lectures: Introduction to motivic categories.

Lecture 1-1 Tom Bachmann
Overview.

Lecture 1-2 Thomas Brazelton
Definition of infinity-categories as quasi-categories, the existence of mapping spaces, interpretation. Presentable infinity categories. Bousfield localization. All of this will have to be presented very tersely, mostly as a reminder.

Lecture 1-3 Thomas Brazelton
The construction of the unstable $\mathbb{A}^1$-homotopy category over a base.
Functorialities, f_*, f^*. Thom spaces. Homotopy purity.

Lecture 1-4 Tom Bachmann
The stable $\mathbb{A}^1$-homotopy category. As $\mathbb{P}^1$-spectra. Suspension functors, "ambidexterity", purity. 6 functors (just construction and properties).

Day 2: 4 Lectures: connectivity, homotopy t-structure and Morel's theorem on stable π_0.

Lecture 2-1 Viktor Burghardt
Morel's S^1-$\mathbb{A}^1$-connectivity theorem. Introduce S^1-spectra, follow Morel's paper "The stable $\mathbb{A}^1$-connectivity theorems".

Lecture 2-2 Viktor Burghardt
Promote the connectivity theorem for $\mathbb{P}^1$-spectra. Introduce the S^1 and $\mathbb{P}^1$ homotopy t-structures.

Lecture 2-3 Fangzhou Jin
The heart of the homotopy t-structure as homotopy modules, following Morel's ICTP notes.

Lecture 2-4 Fangzhou Jin
Introduce the Milnor-Witt K-sheaves. State the main properties without much proof, following the ICTP notes, state Morel's theorems on stable $\pi_{n,n}$, and give a sketch of the proof, following the ICTP notes.

Day 3: 2 Lectures. Motivic cohomology.

Lecture 3-1 Brian Shin
Motivic cohomology via the Voevodsky motivic complexes

Lecture 3-2 Brian Shin
Representing motivic cohomology in $\mathrm{SH}(k)$, following Röndigs-Østvær.

Day 4. 4 Lectures. K-theory, algebraic cobordism, and the slice tower.

Lecture 4-1 Longke Tang
BGL and the unstable representation theorem of Morel-Voevodsky.

Lecture 4-2 Yuchen Wu

KGL and the stable representation theorem.

Lecture 4-3 Niklas Kipp

The construction of MGL. The formal group law. Computation of motivic cohomology of MGL.

Lecture 4-4 Niklas Kipp

Introduction to Voevodsky's slice tower.

Day 5. 4 Lectures. Hopkins-Morel-Hoyois theorem, slices of KGL and MGL.

Lecture 5-1 Klaus Mattis

Statement of the HMH theorem: Introduce the Steenrod algebra and the motivic version as a black box.

Lecture 5-2 Longke Tang

Computation of the homology (rational and mod ℓ) of regular quotients of MGL.

Lecture 5-3 Klaus Mattis

Finish the proof of the HMH theorem.

Lecture 5-4 Tongtong Liang

Compute the slices of KGL and MGL.