

**IWOAT SUMMER SCHOOL 2026:
AN INTRODUCTION TO STABLE HOMOTOPY THEORY AND
BRAVE NEW ALGEBRA**

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ABSTRACT. The purpose of this summer school is to provide a rapid introduction to the foundations of modern stable homotopy theory. We will begin with a treatment of abstract homotopy theory, covering both model categories and $(\infty, 1)$ -categories. Using this base, we will introduce spectra and stable categories from a variety of perspectives. We will then talk about the symmetric monoidal structure on the category of spectra and ring and module objects, both from an operadic and point-set perspective. This will include natural geometric examples, notably Thom spectra. Finally, we will give a streamlined treatment of Bousfield localization. The course will roughly follow the outline of the book Stable categories and structured ring spectra.

Prerequisites: Basic algebraic topology at the level of A Concise Course (including homology, CW-complexes, cofibrations and fibrations, elementary category theory).

Day 1: Abstract homotopy theory.

- (i) Overview of the class.
- (ii) Model categories. The homotopy category, calculus of fractions, model categories, derived functors (Quillen functors), homotopy colimits and limits.
References: Sections 2.2–2.5.
- (iii) Simplicial (model) categories. Simplicial mapping spaces. Quillen’s SM7. Dwyer-Kan simplicial localization.
References: Sections 2.3.5, 2.4.4, 2.6. For SM7, see Section 4.2 of Mark Hovey’s Model Categories.
- (iv) $(\infty, 1)$ -categories. Quasicategories, the homotopy category, mapping spaces, homotopy limits and colimits, comparisons with model and simplicial categories.
References: Sections 2.7–2.10.

Day 2: Spectra and stable categories.

- (i) Diagram spectra; symmetric, orthogonal, excisive functors. The smash product of spectra. Symmetric monoidal model categories.
References: Sections 3.1, 3.5, 3.7–3.8, Section 4.1 for excisive functor, Section 3.3.2 for symmetric monoidal model categories.
- (ii) Γ -spaces. Comparison to diagram spectra, the smash product of Γ -spaces. The model structure on Γ -spaces.
References: Sections 3.4, 3.9, 3.10. Use supplementary reference.

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- (iii) Stable model categories, SM7 revisited. Mapping spectra. Spectral categories. Categories of spectra in model categories.
References: Sections 3.2-3.3, 3.6.
- (iv) Spectra and excisive functors in $(\infty, 1)$ -categories. The symmetric monoidal structure. The universal property of spectra. Stabilization. Stable ∞ -categories. Triangulated categories and t -structures.
References: Sections 4.1-4.7.

Day 3: Rings and modules in stable homotopy theory, part one.

- (i) The model structures on rings and modules in diagram spectra. The positive model structure. Homotopy limits and colimits in rings and commutative rings. Constructions with rings and modules. Change-of-rings and derived categories.
References: Sections 6.1-6.3, Section 3.10.5. Use supplementary reference for homotopy limits and colimits in rings.
- (ii) Morita theory, module categories, bar constructions and cyclic bar constructions. Spectral categories vs. stable ∞ -categories.
References: Sections 1,2 and 4 of Blumberg–Gepner–Tabuada, A universal characterization of higher algebraic K-theory.
- (iii) Operads. Operadic algebras and modules. Model structures on operadic algebras. Rectification. Operadic spaces and Γ -spaces.
References: Sections 5.1–5.11; Sections 3.10.2 for Γ -spaces.
- (iv) Eilenberg-Mac Lane spectra, HM -modules, and chain complexes. Dold-Kan. Algebraic vs. non-algebraic stable categories.
References: Section 6.3; Section 2.4 for the Dold–Kan.

Day 4: Rings and modules in stable homotopy theory, part two.

- (i) Units and Thom spectra. The unit space, spectrum. The defining adjunction. The Thom spectrum as a symmetric monoidal functor.
References: Section 6.4.
- (ii) THH and TAQ ; construction, definition, basic properties. THH as the tensor for commutative rings. Examples of TAQ .
References: Sections 6.6-6.8.
- (iii) Algebraic K -theory; inputs as Waldhausen categories, stable categories. The structural theorem and the universal property. The cyclotomic trace.
References: Section 6.6 for the trace from algebraic K-theory to THH /cyclic-bar context; supplementary references for Waldhausen K-theory, the universal property, and the cyclotomic trace.
- (iv) Picard and Brauer groups, generalized Thom spectra. Examples and computations; the E_∞ -ring structure on ku via bisymmetric monoidal categories.
References: Sections 6.4, 6.5, and 6.9.

Day 5: Bousfield localization and further directions.

- (i) Localization and local objects. Model structure approach via the small object argument. Examples; p -local and p -complete spectra. The rational homotopy category and Sullivan’s theorem.
References: Sections 7.1–7.7; some examples in Sections 7.9–7.10; Section 5.12.

- (ii) Localization in the stable setting, stable ∞ -categories and localization, symmetric monoidal structures.
References: Sections 7.8-7.12.
- (iii) A brief introduction to chromatic localizations and the chromatic perspective on the stable category.
References: Sections 7.1 and 7.9; supplementary chromatic homotopy references are recommended.
- (iv) Visions of spectral algebraic geometry.
References: Chapter 8.

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